

Math 7 Study Sheet (Chapter 1)
Ratios and Proportions

Name _____
Block _____ Date _____

Key

Test on Chapter 1 is Tuesday, December 12

Review your notes, quizzes, homework problems, and handouts for this chapter.

Be able to do the following:

- *Write and simplify a ratio.
- *Calculate unit rate and use it to make comparisons.
- *Write and simplify a complex fraction as it applies to unit rates.
- *Convert units using dimensional analysis.
- *Graph a relationship and tell whether or not it is proportional.
- *Write and solve a proportion using at least three methods.
- *Find the constant rate of change of a relationship.
- *Find the slope of a line on a graph of a relationship or from a table
- *Write an equation using the constant of proportionality.

Vocabulary

- | | | |
|--|----------------------|------------------------------|
| *Rate | *Coordinate plane | *Constant rate of change |
| *Unit Rate | *Quadrants | *Slope |
| *Complex fraction | *x and y-coordinates | *Direct variation |
| *Measurement equivalent (name for one) | *x and y-axes | *Constant of variation |
| *Dimensional analysis | *origin | *Constant of proportionality |
| *Proportional/nonproportional | *proportion | |
| *Equivalent ratios | *cross-products | |

Solve the following problems AFTER reviewing your notes, homework, quizzes and other handouts for this chapter. You MUST SHOW ALL STEPS and WORK for each problem unless it says NWN (No Work Needed)

1) Find the unit rate in miles per hour for this situation. In 4.4 hours, a car travels 220 miles.

$$\frac{\text{mi}}{\text{hr}} \frac{220}{4.4} = 50 \text{ mi/hr}$$

2) Find the unit rate in beats per minute.

Sara's heart beats 30 times in 45 seconds. $45 \text{ sec} = \frac{3}{4} \text{ min}$

$$\frac{\text{beats}}{\text{min}} \frac{30}{\frac{3}{4}} = 30 \div \frac{3}{4} = \frac{30 \cdot 4}{3} = \frac{120}{3} = 40 \text{ beats/min}$$

3) Write these ratios in simplest form:

A) 50:225 $\frac{50 \div 25}{225 \div 25} = \frac{2}{9}$

B) 32 to 4 $\frac{32}{4} = \frac{8}{1}$

C) $\frac{15 \div 3}{27 \div 3} = \frac{5}{9}$

4) In Jen's math class there are two lefties, 17 righties, and 1 person who is ambidextrous. Find these ratios:

A) lefties to all students

$$\frac{\text{left}}{\text{All}} \frac{2}{20} = \frac{1}{10}$$

B) righties and lefties to ambidextrous

$$\frac{\text{right} + \text{left}}{\text{ambid.}} \frac{17+2}{1} = \frac{19}{1}$$

5) Simplify these ratios:

A) $\frac{10 \text{ feet}}{18 \text{ inches}} \frac{120 \cancel{\text{ in}}}{18 \cancel{\text{ in}}} = \frac{60}{9} = \frac{20}{3}$

B) $\frac{125 \text{ centimeters}}{25 \text{ meters}} \frac{125 \cancel{\text{ cm}}}{2500 \cancel{\text{ cm}}} = \frac{125 \div 25}{100} = \frac{5}{100} = \frac{1}{20}$

6) Show how to find the slope of the line that goes through these two points. Give answer as a simplified ratio and as a decimal. (4,6) (10,15)

$$\frac{\Delta y}{\Delta x} = \frac{15-6}{10-4} = \frac{9}{6} = \frac{3}{2}$$

7) Simplify this complex fraction. Remember to show steps.

$$\frac{2\frac{1}{4}}{\frac{5}{8}} = 2\frac{1}{4} \div \frac{5}{8}$$

$$\frac{9}{4} \cdot \frac{8^2}{5}$$

$$\frac{18}{5}$$

$$\frac{3\frac{3}{5}}$$

8) Show how to use complex fractions to write $4\frac{3}{8}\%$ as a simplified fraction.

$$\frac{4\frac{3}{8}}{100} = 4\frac{3}{8} \div 100$$

$$\frac{35}{8} \cdot \frac{1}{100}$$

$$\frac{7}{160}$$

9) Jon can run $3\frac{5}{8}$ miles in $\frac{7}{8}$ of an hour. How many miles per hour can he run?

$$\frac{\text{mi}}{\text{hr}} \cdot \frac{3\frac{5}{8}}{\frac{7}{8}} = 3\frac{5}{8} \div \frac{7}{8}$$

$$= \frac{29}{8} \cdot \frac{8}{7}$$

$$= \frac{29}{7}$$

$$= 4\frac{1}{7} \text{ mi/hr}$$

10) Ben was driving in traffic for $3\frac{1}{4}$ hours. He drove a total of 130 miles. Find his average miles per hour for this time.

$$\frac{\text{mi}}{\text{hr}} \cdot \frac{130}{3\frac{1}{4}} = 130 \div 3\frac{1}{4}$$

$$= \frac{130}{1} \div \frac{13}{4}$$

$$= \frac{130}{1} \cdot \frac{4}{13}$$

$$40 \text{ mi/hr}$$

11) Use dimensional analysis to convert 68 quarts to gallons.

$$\frac{68 \text{ qt}}{1} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} = \frac{68}{4} =$$

$$17 \text{ gal}$$

12) Use dimensional analysis to convert 8 miles/min to feet/min.

$$\frac{8 \text{ mi}}{1 \text{ min}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = 8(5280)$$

$$42,240 \text{ ft/min}$$

13) Use dimensional analysis to convert 20 cm/sec to ___ m/hour.

$$\frac{20 \text{ cm}}{1 \text{ sec}} \cdot \frac{1 \text{ m}}{100 \text{ cm}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$$

$$\frac{20 (60) (60)}{100}$$

$$720 \text{ m/hr}$$

14) Use dimensional analysis to convert 63 yards/sec to ___ inches/minute

$$\frac{63 \text{ yd}}{1 \text{ sec}} \cdot \frac{36 \text{ in}}{1 \text{ yd}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} =$$

$$63 (36) (60)$$

$$136,080 \text{ in./min}$$

15) Can the ratios $\frac{6}{10}$ and $\frac{9}{15}$ be used to make a proportion? Explain your reasoning. Remember to explain what a proportion is.

$$6 \cdot 15 = 90 \quad 10 \cdot 9 = 90$$
$$\frac{6}{10} = \frac{9}{15}$$

Check cross products
A proportion is an equation in which cross products are equal.
In this case they are both 90.

16) Solve this proportion. Show the method you use.

$$\frac{2}{9} = \frac{n}{36}$$

$$n = 8$$

17) (NWN) Write a word ratio and proportion that can be used to solve this problem. Do not solve. Jen knows 3 cups of fruit feeds 5 people. There will be 100 people at the party. How many cups of fruit will she need?

$$\frac{\text{c. fruit}}{\text{\# people}} = \frac{3}{5} = \frac{n}{100}$$

18) (NWN) Write a word ratio and proportion that can be used to solve this problem. Do not solve. A machinist can produce 114 parts in 6 minutes. At this rate, how many parts can the machinist produce in 15 minutes?

$$\frac{\text{parts}}{\text{min}} = \frac{114}{6} = \frac{n}{15}$$

19) In this proportion what is the value of the cross products?

Explain how you found the value.

$$\frac{129.6}{2.5} = \frac{90}{7.2} \quad 2.5(90) = 225$$

One is 129.6
18(7.2)

the other is 225
2.5(90)

Therefore this is not
a proportion.

22) Show how to find the slope of the line that goes through the points (6,4) and (8,7).

$$\frac{\Delta y}{\Delta x} = \frac{7-4}{8-6} = \frac{3}{2}$$

20) (NWN) Name the quadrant or axis in which each of the following points are located.

A) (0, -5) \vdash y axis

B) (-6, 10) \vdash Q II

C) (2, 4) \vdash Q I

D) (5, 0) \vdash x axis

E) (-9, -3) \vdash Q III

21) (NWN) The equation $y = 12x$ represents the amount of money (y) that Ted earns for (x) hours of work.

A) What is the value of the constant of proportionality? 12

B) Explain what that value represents.
Ted earns \$12/hour

C) What other names have we learned for the constant of proportionality?
unit rate, slope,
constant of proportionality

23) The equation $y = 28x$ represents the amount of money (y) Tracy will spend for x dresses.

A) (NWN) Identify the constant of proportionality. 28

B) (NWN) Explain what it means in this situation.

She spends \$28 per dress.

C) Show how to use the equation to find what it would cost Tracy for 4 dresses.

$$y = 28x$$
$$y = 28(4)$$
$$y = 112 \quad \text{\$112}$$

24) Write the equation to represent this proportional relationship. Show how you came up with your equation.

Let y = the number of miles traveled
Let x = the number of hours it takes to travel y miles

In 3 hours, Tanya travels 165 miles and in 10 hours she travels 550 miles.

$$(x, y) \quad (3, 165) \quad (10, 550)$$

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{550 - 165}{10 - 3} = \frac{385}{7} = 55$$

$y = kx$ since it is proportional

$$y = 55x$$

25) We have learned three ways to solve proportions: equivalent ratios, simplify one ratio first, and show algebraic steps using cross products. Choose the best proportion to be used for each method, copy it in the appropriate space below and solve using that method. Round to the nearest hundredth if necessary (numbers not nice).

$$\frac{19}{n} = \frac{16}{32}$$

$$\frac{7}{t} = \frac{17}{22}$$

$$\frac{x}{15} = \frac{4}{5}$$

Equivalent Ratios

$$\frac{x}{15} = \frac{4}{5}$$

x3

$$x = 12$$

Simplify one ratio first

$$\frac{19}{n} = \frac{16}{32}$$

$$\frac{19}{n} = \frac{1}{2}$$

$$\frac{19}{n} = \frac{1}{2}$$

x19

$$n = 38$$

Algebraic steps using crossproducts

$$\frac{7}{t} = \frac{17}{22}$$

$$7(22) = 17t$$

$$154 = 17t$$

$$\frac{154}{17} = \frac{17t}{17}$$

$$9.05882... = t$$

$$t \approx 9.06$$

26) Show how to determine if a person's height is proportional to his age.

Age (yr)	Height (in.)
9	54
10	56
11	58
12	60

$\frac{\text{height}}{\text{age}}$

$$\frac{54}{9} = 6$$

$$\frac{56}{10} = 5.6$$

$$\frac{58}{11} = 5.27$$

$$\frac{60}{12} = 5$$

No because ratio of height to age is not constant

27) Show how to determine if the temperature is proportional to the time?

Time (h)	Temperature (°C)
0	0
4	3
8	6
12	9

unit rate

$$\frac{3^{\circ}\text{C}}{4 \text{ hr.}}$$

$$0.75 \text{ } ^{\circ}\text{C/hr}$$

28) Solve this proportion showing correct algebraic steps.

$$\frac{4.5}{n} = \frac{15}{2.8}$$

$$(4.5)(2.8) = 15 \cdot n$$

$$12.6 = 15n$$

$$\frac{12.6}{15} = \frac{15n}{15}$$

$$0.84 = n$$

29) Show how to find the best buy based on unit price for these tubes of toothpaste.

Crest \$3.50 for 16 ounces *

Colgate \$3.00 for 12 ounces

Aquafresh \$1.99 for 6 ounces

cheapest per oz

	Crest	Colgate	Aqua
cost	3.50	3.00	1.99
oz	16	12	6
	.21875	\$0.25/oz	0.3316
	\$0.22/oz		\$0.33/oz.

30) Fun Center rents popcorn machines for \$20 per hour. In addition to the hourly charge, there is a rental fee of \$32. Complete the chart for the cost of renting a popcorn machine at Fun Center. Then determine if the total cost is proportional to the number of hours the machine is rented.

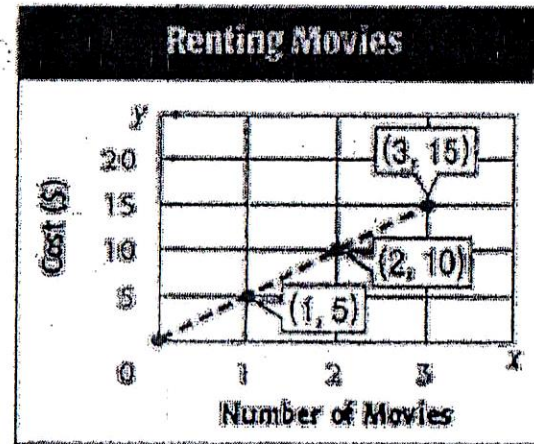
Hours	1	2	3	4
Cost (\$)	\$52	\$72	\$92	\$112

$\frac{\text{Total Cost}}{\text{\# hours}}$	$\frac{52}{1}$	$\frac{72}{2}$	$\frac{92}{3}$	$\frac{112}{4}$
	(52)	(36)	(30.6)	(28)

No ratios of total cost to # hours is not constant

$$52 \neq 36 \neq 30.6 \neq 28$$

31) This graph shows the cost of renting movies.



A) Show how to find the constant rate of change for the graph. (3, 15) (1, 5)

$$\frac{\Delta y}{\Delta x} = \frac{15-5}{3-1} = \frac{10}{2} = 5 \quad \text{\$5/movie}$$

B) Is the cost proportional to the number of movies?

Explain. Yes graph is straight line that goes through origin

C) Show how to find the slope of this line.

See part A (or) look at unit rate (1, 5) on graph since its proportional (1, 5)

D) Show how to find the unit rate (cost per movie)

$$\frac{\text{cost}}{\text{movie}} = \frac{5}{1} = 5 \quad \frac{10}{2} = 5 \quad \frac{15}{3} = 5 \quad \text{\$5/movie}$$

E) Show how to find the constant of proportionality.

Same as slope $k=5$

F) Write the equation of the line. Explain how you wrote it.

$y = kx$ for proportional relationships

$$y = 5x$$

$$k=5$$

32) Show how to find the best buy based on unit price (remember this means you need to correctly find the unit rate rounded to the nearest cent for each one) for these packages of Pop Tarts.

Regular size \$3.50
(6 Pop Tarts per box)

cost per pop tart
 $3.50 \div 6$
 $.5833$ $\approx 0.58/\text{pop tart}$

Family size \$5.50 *
(12 Pop Tarts per box)

$5.50 \div 12$
 $.45833$ $\approx 0.46/\text{pop tart}$

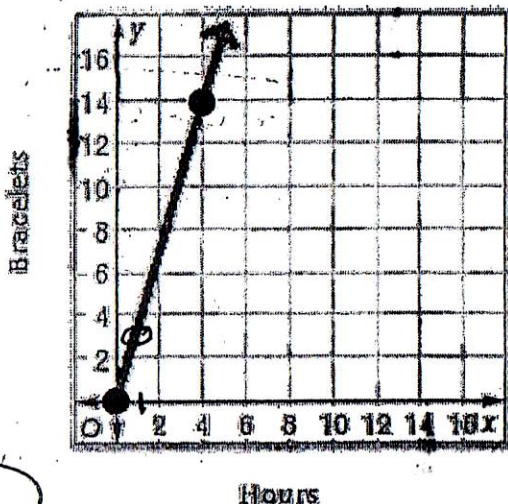
Super size \$12.00
(4 packages of
6 Pop Tarts per box)

$12 \div 24$
 $.5$ $\approx 0.50/\text{pop tart}$

Best Buy is Family size.

33) The number of bracelets Robin makes varies directly as the number of hours she works.

A) Use the graph to find how many bracelets she makes in 4 hours and how many bracelets she makes in zero hours. Write this information as two ordered pairs.



$(0,0)$ $(4,14)$

B) Show how to find the slope of the line.

$$\frac{\Delta y}{\Delta x} = \frac{14-0}{4-0} = \frac{14}{4} = \frac{7}{2}$$

C) Write the equation of this line using the variables x and y.

$y = kx$
 $y = \frac{7}{2}x$

D) Show how to find the unit rate and explain how to check it on the graph.

$\frac{7}{2} = 3.5$
 makes 3.5 bracelets per hour

On graph find $x=1$
 Then look at y value. It looks like it is between 2 and 4 but closer to 4 so 3.5 makes sense.

34) Use the graph to answer these questions.

A) Explain what the point (4,12) means on the graph below. There are 12 feet in 4 yards.

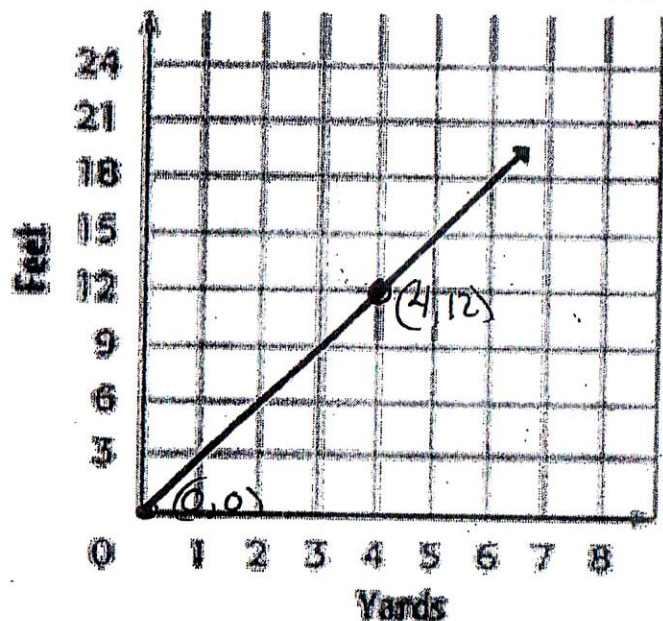
B) How can you tell from the graph that the number of feet is proportional to the number of yards? The graph is a straight line that goes through origin.

C) Write the equation of this line using the variables x and y from the coordinate grid.

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{12-0}{4-0} = \frac{12}{4} = 3$$

$$y = kx$$

$$y = 3x$$



35) The table below shows the number of apples per basket at a farmer's market. (x,y) (# baskets, # apples)

Baskets	2	4	6	8
# of apples	10	20	30	40

A) Graph the data.

B) Show how to find the slope of the line.

Choose 2 points (4,20) (8,40)

$$\frac{\Delta y}{\Delta x} = \frac{40-20}{8-4} = \frac{20}{4} = 5$$

C) Explain what the slope represents.

There are 5 apples per basket

D) Write the equation of the line.

Let b = # of baskets

Let a = # of apples

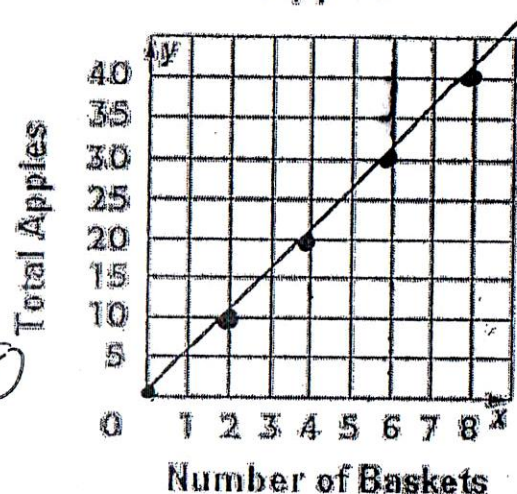
$$y = kx$$

↓

$$b = ka$$

$$b = 5a$$

Apples



36) **Multiple Choice:** Explain the work you did to choose your answer.

Which table shows that the rate of change is 30 cm for every meter?

A.

m	cm
3	90
4	120
5	150

$$\frac{\text{cm}}{\text{m}} \quad \frac{90}{3} = 30$$

$$\frac{120}{4} = 30$$

$$\frac{150}{5} = 30$$

B.

m	cm
90	3
120	4
150	5

C.

m	cm
4	30
5	60
6	90

D.

m	cm
1	50
2	60
3	70

37)

This is a table showing the time and distance Lou ran. How fast did Lou run?

Time (seconds)	0	5	10	15	20
Distance (meters)	0	22.5	45	67.5	90

$$\frac{\text{m}}{\text{sec}} \quad \frac{22.5}{5} = 4.5 \text{ m/sec}$$

$$\frac{45}{10} = 4.5$$

$$\frac{67.5}{15} = 4.5$$

$$\frac{90}{20} = 4.5$$

38) What does each of the following refer to? Then label them on the graph.

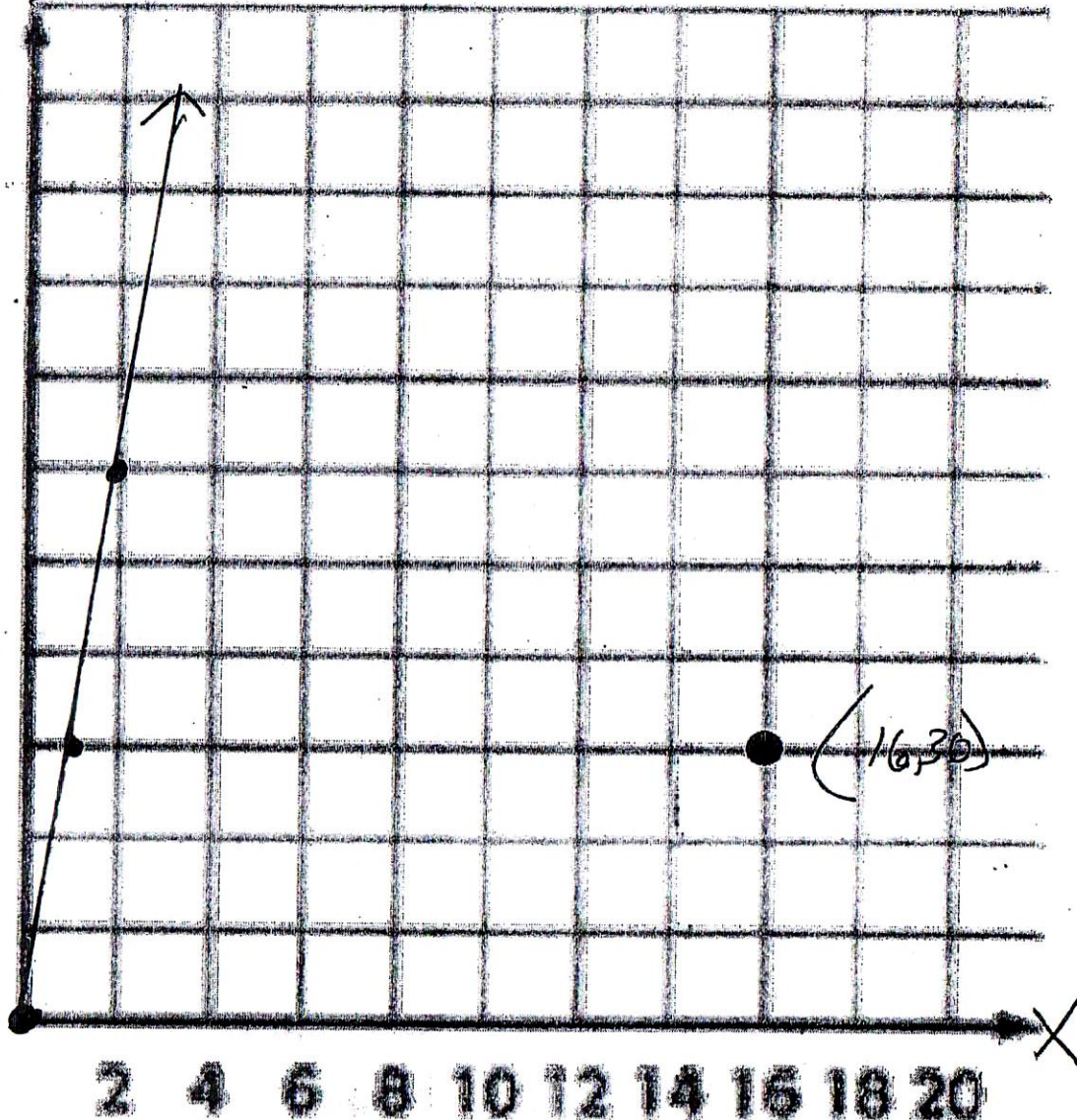
- a) Origin $(0,0)$
- b) x-axis *horizontal axis*
- c) y-axis *vertical axis*
- d) x-coordinate *16*
(use point provided)
- e) y-coordinate *30*
(use point provided)
- f) Ordered Pair
(use point provided)

$(16,30)$

g) Draw a line in which y is proportional to x AND the line has a slope of 30 $(1,30)$.

*should be on graph.
Slope = unit rate*

100
90
80
70
60
50
40
30
20
10
0



39) Use the graph to answer these questions.

A) (NWN) The number of laps is proportional to the number of minutes.

B) Show how to find the constant rate of change for this graph. Choose 2 points (10, 5) (20, 10)

$$\frac{\Delta y}{\Delta x} = \frac{10-5}{20-10} = \frac{5}{10} = \frac{1}{2} \quad 0.5 \text{ laps/min}$$

C) What is the meaning of the point (20, 10) ?

Go 10 laps in 20 minutes

