Math 7
Converting Unit Rates
Dimensional Analysis
Notes (Section 1-3)

Review:

*Identity Property of Multiplication

We can multiply any number by 1 without changing its value.

250 x 1 = 250

-7 x 1 = -7

4.1 x $\frac{5}{5}$ = 4.1

78 x $\frac{3+4}{8-1}$ = 78

4 x $\frac{1 \text{ foot}}{12 \text{ inches}}$ = 4

$8 \cdot \frac{1 \text{ meter}}{100 \text{ cm}} = 8$

1 m = 100 cm

Names for 1 using measurement equivalents

$1 = \frac{1 \text{ m}}{100 \text{ cm}} = \frac{100 \text{ cm}}{1 \text{ m}} = \frac{1 \text{ yard}}{36 \text{ in}} = \frac{36 \text{ in}}{1 \text{ yard}} = 1$

Dimensional analysis is the process of including units of measure when computing. We start with the original measurement or rate and multiply by names for one based on measurement equivalents.

Round to nearest tenth

Example Change 8 in/ min to _____ft/min

Goal: $\frac{ft}{min}$

$\frac{8 \text{ in}}{1 \text{ min}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{8(1)}{1(12)} = \frac{8}{12}$

$= 0.6666\ldots \approx 0.7 \text{ ft/min}$

12 in = 1 ft

Example Change 36 in/sec to _____ft/min

$\frac{36 \text{ in}}{1 \text{ sec}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{60 \text{ sec}}{1 \text{ min}}$

$\frac{ft}{min}$

$\frac{36(1)(60)}{(1)(12)(1)} = \frac{2160}{12}$

12 in = 1 ft

60 sec = 1 min

180 ft/min

Jon runs 1500 meters in 4 minutes. How many millimeters does he run per second?

*Write the original rate

*Write your GOAL for units

*Choose one of the original units. Look at goal. Use the measurement equivalent to multiply by 1. Simplify units.

*Repeat process until goal is met.

*Record the numbers in both numerators and denominators

*Multiply #'s in numerator
Multiply #'s in denominator

*Divide:
Numerator ÷ Denominator

*Round answer when necessary and label

$\frac{1500 \text{ m}}{4 \text{ min}} \cdot \frac{1000 \text{ mm}}{1 \text{ m}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$

$\frac{mm}{sec}$

$\frac{1500(1000)(1)}{4(1)(60)}$

$\frac{1500000}{240} = 1{,}500{,}000 \div 240$

6250 mm/sec

1000 mm = 1 m

1 min = 60 sec

Jon runs 5 kilometers in 20 minutes. How many millimeters does he run per second?

*Write the original rate

*Write your GOAL for units

*Choose one of the original units. Look at goal. Use the measurement equivalent to multiply by 1. Simplify units.
*Repeat process until goal is met.

*Record the numbers in both numerators and denominators

*Multiply #'s in numerator
Multiply #'s in denominator

*Divide:
Numerator ÷ Denominator

*Round answer when necessary and label

See last page for work

Dimensional Analysis

The next two pages of the flipchart are my attempt to show step by step instructions for anyone helping one of my students who has never done dimensional analysis. It is a difficult process for some students but it will be used in science classes over the next few years.

Jon runs 800 meters in 2 minutes. How many millimeters does he run per second?

Write original rate with units $\frac{800m}{2\text{min}}$

Write GOAL for units → GOAL $\frac{\text{mm}}{\text{sec}}$

Choose one of original units
Write a name for one using measurement equivalents
1 min = 60 sec → $\frac{1\text{min}}{60\text{sec}}$

Multiply by 1 and simplify measurement units $\frac{800m}{2\text{min}} \times \frac{1\text{min}}{60\text{sec}}$

Choose name for one 1 m = 1000 mm → $\frac{1000\text{mm}}{1m}$

Multiply by 1 and simplify measurement units $\frac{800m}{2\text{min}} \times \frac{1\text{min}}{60\text{sec}} \times \frac{1000\text{mm}}{1m}$

Simplify units until goal is reached $\frac{800\text{m}}{2\text{min}} \times \frac{1\text{min}}{60\text{sec}} \times \frac{1000\text{mm}}{1\text{m}}$

Record the numbers in both numerators and denominators $\frac{(800)\,(1)(1000)}{2\,(60)}$

Multiply #'s in numerator
Multiply #'s in denominator $\frac{800{,}000}{120}$

Divide:
Numerator ÷ Denominator 800,000 ÷ 120

Round answer when necessary and label 6666.666666...

6666.7 mm/sec

6 Liters = _____ mL

$\frac{6 \text{ Liters}}{1} \cdot \frac{1000 \text{ mL}}{1 \text{ Liter}}$ → mL

$\frac{6(1000)}{(1)(1)} = 6000 \text{ mL}$

1 liter = 1000 mL

500 pounds = _____ tons

$\frac{500 \text{ lbs}}{1} \cdot \frac{1 \text{ ton}}{2000 \text{ lbs}}$ → tons

$\frac{500(1)}{1(2000)} = \frac{500}{2000} = 0.25 \text{ tons}$

1 ton = 2000 lbs

45 feet = _____ yards

$\frac{45 \text{ ft}}{1} \cdot \frac{1 \text{ yd}}{3 \text{ ft}}$ → yd

$\frac{45(1)}{(1)(3)} = \frac{45}{3} = 15 \text{ yd}$

3 ft = 1 yd

6.8 meters = _____ centimeters

$\frac{6.8 \text{ m}}{1} \cdot \frac{100 \text{ cm}}{1 \text{ m}}$ → cm

$(6.8)(100) = 680 \text{ cm}$

100 cm = 1 m

Liz is averaging 60 miles per hour on her trip. How many feet per minute is this?

$\frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}}$ → $\frac{\text{feet}}{\text{min}}$

$\frac{60(1)(5280)}{(1)(60)(1)} = \frac{316800}{60}$

5280 ft/min

1 mile = 5280 ft

1 hour = 60 min

Sara's pool is draining at a rate of 20 quarts per minute. How many gallons per minute is this?

$\frac{20 \text{ qts}}{1 \text{ min}} \cdot \frac{1 \text{ gal}}{4 \text{ qts}}$ → $\frac{\text{gal}}{\text{min}}$

$\frac{20(1)}{(1)(4)}$

$\frac{80}{4} = 5 \text{ gal/min}$

4 qt = 1 gal

Raindrops can fall as fast as 20 miles per hour. How many feet per minute is this?

$$\frac{20\ \text{mi}}{1\ \text{hr}} \cdot \frac{5280\ \text{ft}}{1\ \text{mi}} \cdot \frac{1\ \text{hr}}{60\ \text{min}} \qquad \left[\frac{\text{ft}}{\text{min}}\right]$$

$$\frac{20(5280)(1)}{(1)(1)(60)}$$

$$\frac{105600}{60} = 1760\ \text{ft/min}$$

$$\left[\frac{1\ \text{mi} = 5280\ \text{ft}}{1\ \text{hr} = 60\ \text{min}}\right]$$

Jon

$$\frac{5\ \text{km}}{20\ \text{min}} \cdot \frac{1000\ \text{m}}{1\ \text{km}} \cdot \frac{1000\ \text{mm}}{1\ \text{m}} \cdot \frac{1\ \text{min}}{60\ \text{sec}} \qquad \left[\frac{\text{mm}}{\text{sec}}\right]$$

$$\frac{(5)(1000)(1000)(1)}{(20)(1)(1)(60)}$$

$$\frac{5{,}000{,}000}{1200}$$

4166.666

4166.7 mm/sec

km → m → mm

$$\left[\frac{1\ \text{km} = 1000\ \text{m}}{1000\ \text{mm} = 1\ \text{m}}\right]$$

1 min = 60 sec