

Accelerated Math Notes
 (from Math 7 textbook
 Section 1.7 and 1.8)
Constant Rate of Change and Slope

A **rate of change** is a rate that describes how one quantity changes in relation to another. When a relationship is linear (forms a straight line), it has a constant rate of change.

This change can be found by using a table or a graph. On a graph, this change is also known as the **slope** of a line.

Vocabulary Start-Up

A **rate of change** is a rate that describes how one quantity changes in relation to another. In a linear relationship, the rate of change between any two quantities is the same. A linear relationship has a **constant rate of change**.

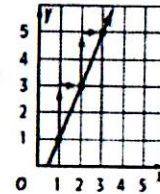
Another name for this is:

Key Concept → Slope

Slope is the rate of change between any two points on a line.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} \begin{matrix} \leftarrow \text{vertical change} \\ \leftarrow \text{horizontal change} \end{matrix}$$

$$= \frac{2}{1} \text{ or } 2$$



The table below shows the amount of money a booster club earns washing cars for a fundraiser.

You can use a table to find the Constant Rate of Change

Cars Washed	
Number	Money (\$)
5	40
10	80
15	120
20	160

Handwritten annotations: Brackets on the left indicate a change of +5 in the number of cars for each row. Brackets on the right indicate a change of +40 in the money earned for each row.

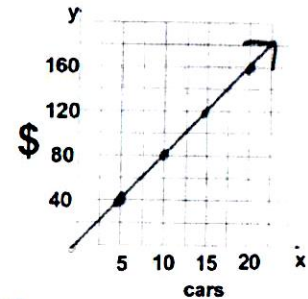
$\frac{\text{Change in money}}{\text{Change in cars}} = \frac{40}{5} = 8/\text{car}$

Find ratios $\frac{40}{5}$ $\frac{80}{10}$ $\frac{120}{15}$ $\frac{160}{20}$
 all equal $\frac{8}{1}$

Proportional since cost per car always the same

You can use a graph and find the constant rate of change. This is also known as the slope.

Choose 2 points
 $(20, 160)$ $(5, 40)$



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{160 - 40}{20 - 5}$$

$$= \frac{120}{15}$$

$$= 8$$

means change in y / change in x

We can find the slope of a line by choosing two points and using this formula.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{matrix} (0, 5) & (2, 1) \\ x_1, y_1 & x_2, y_2 \end{matrix} \quad \text{OR} \quad \begin{matrix} (0, 5) & (2, 1) \\ x_2, y_2 & x_1, y_1 \end{matrix}$$

$$\begin{aligned} \text{slope} &= \frac{1-5}{2-0} \\ &= \frac{-4}{2} \\ &= \textcircled{-2} \end{aligned}$$

$$\begin{aligned} \text{slope} &= \frac{5-1}{0-2} \\ &= \frac{4}{-2} \\ &= \textcircled{-2} \end{aligned}$$

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Find slope of line through:

$$\begin{matrix} (2, -1) & \text{and} & (5, 8) \\ x_1, y_1 & & x_2, y_2 \end{matrix}$$

$$\begin{aligned} \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8 - (-1)}{5 - 2} \\ &= \frac{9}{3} \\ &= \textcircled{3} \end{aligned}$$

$$\begin{matrix} (-6, 1) & (4, -8) \\ x_1, y_1 & x_2, y_2 \end{matrix}$$

$$\begin{aligned} \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - 1}{4 - (-6)} \\ &= \frac{-9}{10} \end{aligned}$$

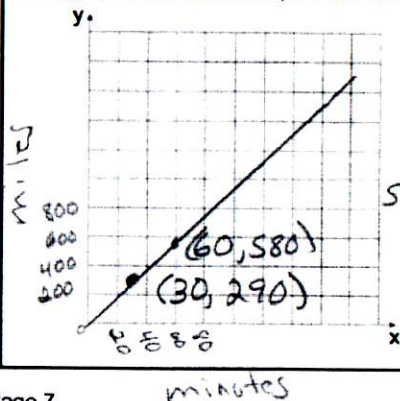
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The table shows the number of miles a plane traveled while in flight. Use the information to find the approximate constant rate of change in miles per minute.

X	Time (min)	30	60	90	120
Y	Distance (mi)	290	580	870	1160

$$\begin{matrix} \text{miles} & y & 290 & 580 & 870 & 1160 \\ \text{min} & x & 30 & 60 & 90 & 120 \\ & & 9.6 & 9.6 & 9.6 & 9.6 \end{matrix}$$

Now graph the relationship. Find the slope.



$$\begin{aligned} \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{580 - 290}{60 - 30} \\ &= \frac{290}{30} \\ &= 9.6 \text{ or } \frac{29}{3} \end{aligned}$$

Since it is a proportional relationship, the constant ratio is the unit rate and constant rate of change $\textcircled{9.6 \text{ miles/min}}$

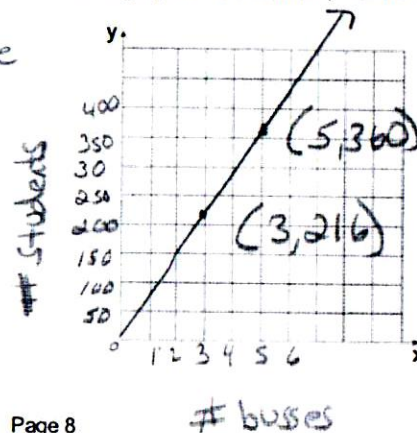
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The table shows the number of students that buses can transport. Use the table to find the constant rate of change in students per school bus.

x	Number of Buses	2	3	4	5
y	Number of Students	144	216	288	360

$$\frac{\Delta y}{\Delta x} = \frac{72}{1} = \textcircled{72 \text{ students/bus}}$$

Now graph the relationship. Find the slope.

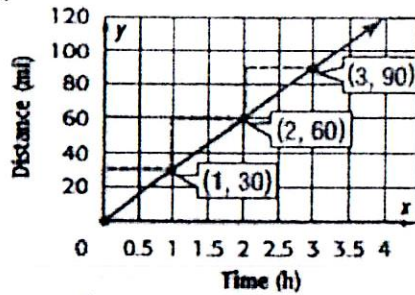


$$\begin{aligned} \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{360 - 216}{5 - 3} \\ &= \frac{144}{2} \\ &= \textcircled{72} \end{aligned}$$

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Use the graph to find the constant rate of change in miles per hour while driving through the city.

30 miles/hour
 (1, 30)
 Unit rate



If its proportional then unit rate is constant rate of change

Explain what the point (1, 30) represents on the graph.

In 1 hour you go 30 miles.

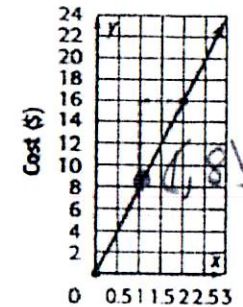
The table and graph both show the hourly charge to rent a bicycle at two different stores. Which store charges more per bicycle? Explain by finding each constant rate of change.

Pedals Rentals	
Time (hour)	Cost (\$)
2	24
3	36
4	48

$$\frac{24}{2} = 12/\text{hour}$$

(0,0) (2,16)
 slope = $\frac{16-0}{2-0} = 8$

Super Cycles



\$8/hour.
 Look at point (1,8) to get unit rate

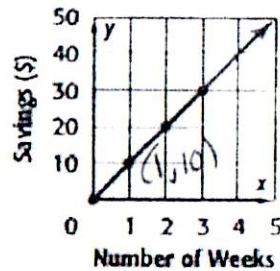
The table and graph both show the amount of money that Mi-Ling and Daniel save each week. Who saves more per week? Explain by finding each constant rate of change.

Mi-Ling's Savings	
Time (weeks)	Savings (\$)
2	\$30
3	\$45
4	\$60

Choose 2 points
 (2,30) (4,60)
 x_1, y_1 x_2, y_2

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 30}{4 - 2} = \frac{30}{2} = 15/\text{week}$$

Daniel's Savings



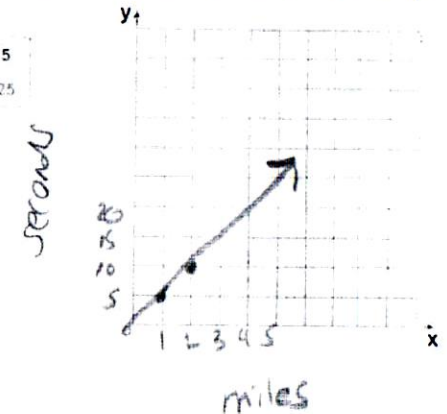
Proportional so look at point (1,10)
 \$10/week

* Mi-Ling

The table shows the relationship between the number of seconds y it takes to hear thunder after a lightning strike and the miles x you are from lightning. Graph the data and find the slope.

Miles (x)	0	1	2	3	4	5
Seconds (y)	0	5	10	15	20	25

$$\frac{x}{y} = \frac{4}{5}$$

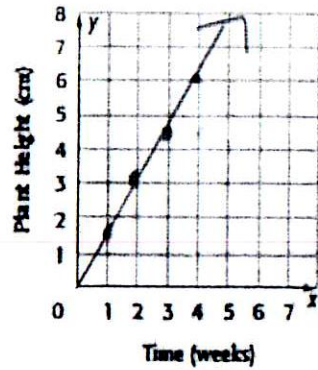


What does this slope represent? 5

It takes 5 seconds to hear thunder 1 mile away

Graph the data about plant height for a science fair project. Then find the slope of the line.

Week	Plant Height (cm)
1	1.5
2	3
3	4.5
4	6



$$\text{Slope} = 1.5$$

use unit rate point (1, 1.5)

What does this slope represent?

Plant grows 1.5cm each week