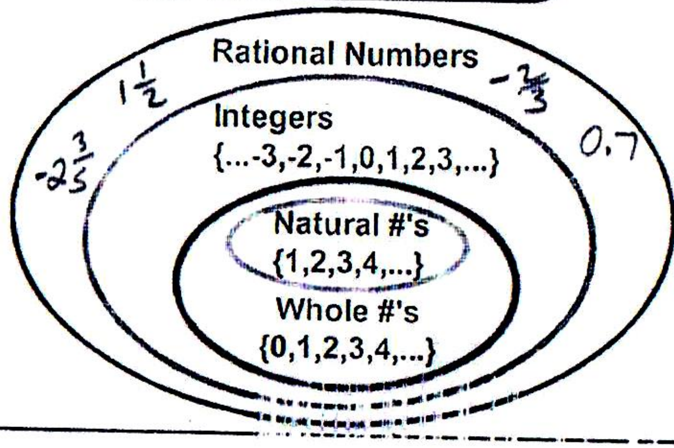
Accelerated Math Notes
(Lesson 3.2)
Rational Numbers



A rational number is a number that can be written as a ratio in the form $\frac{a}{b}$ where a and b are integers and b ≠ 0

We can verify these are rational numbers by using the definition.

$\frac{-3}{4} = -\frac{3}{4}$	$4\frac{2}{3} = \frac{14}{3}$	$0.5 = \frac{5}{10}$
a = -3 b = 4	a = 14 b = 3	a = 5 b = 10

If a number is a rational number, it can be written either as a repeating decimal or a terminating decimal.

$\frac{3}{2}$ is rational because 1.5 is terminating decimal.

$\frac{1}{3}$ is rational because $0.\overline{3}$ is a repeating decimal.

$\sqrt{2}$ is NOT rational (It is irrational) because the decimal 1.4142136 continues but never repeats in a pattern that we could identify with bar notation.

Other irrational #'s

π $\sqrt{11}$

Identify all sets to which each number belongs.

Rational (Q) Irrational # (Z) Integer (I) Whole # (W) Natural # (N)

- $\sqrt{16} = 4$ — Q I W N
- π — Z
- -1.4 — Q
- $\frac{5}{12}$ — Q
- $3\frac{4}{5}$ — Q
- $\sqrt{3}$ — Z
- 48 — Q N I W
- -12 — Q I

When we use rational numbers we often need to find an equivalent form of the number to understand the situation.

Payton has a "two ninety six" batting average.

$.296 = \frac{296}{1000}$ $\frac{\text{hits}}{\text{at bats}}$

The scale at the deli counter says 0.7 and Josh asked for three fourths of a pound of ham.

0.75 scale is a little less than $\frac{3}{4}$

Ways to compare rational numbers:

*Use a 0 $\frac{1}{2}$ 1 benchmark chart

*Write all numbers as decimals

*Write all numbers as fractions with like denominators

*Use a combination of the above strategies

Write these numbers in order from smallest to largest:

0.006 $\frac{73}{75}$ 0.57 $\frac{7}{500}$ $\frac{5}{16}$

Close to 0	Close to $\frac{1}{2}$	Close to 1
1 $.006 = \frac{6}{1000}$	3 $\frac{5}{16}$ 4 $.57$	5 $\frac{73}{75}$
2 $\frac{7}{500} = \frac{14}{1000}$		

Write these numbers in order from smallest to largest: $\frac{3}{5}$ 0.62 0.007 $\frac{5}{9}$ $\frac{3}{50}$

Close to 0	Close to $\frac{1}{2}$	Close to 1
2 $\frac{3}{50} = .06$	5 $.62$	
1 $.007$	3 $\frac{5}{9} = .\overline{5}$	
	4 $\frac{3}{5} = .6$	

Less than (<), Greater than (>), or Equal (=) ???

Verify by using two different methods.

$\frac{2}{3}$ > $\frac{13 \times 5}{20 \times 5}$

1. $.\overline{6}$ > $.65$
2. $\frac{40}{60}$ > $\frac{39}{60}$

0.15 < $\frac{1}{6}$

$\frac{15}{100}$ $\frac{1}{6}$

$\frac{90}{600}$ $\frac{100}{600}$

$.15$ $.1\overline{6}$

Less than (<), Greater than (>), or Equal (=) ???

Verify by using at least two different methods.

$\frac{4}{9}$ < 0.49

1. $.444\ldots$ $.49$
2. $\frac{4}{9} = \frac{400}{900}$ $\frac{441}{900}$

$\frac{3}{4}$ > $\frac{2}{3}$

1. 0 | $\frac{1}{2}$ | 1 ; $\frac{2}{3}$ $\frac{3}{4}$
2. $.75$ $.\overline{6}$
3. $\frac{9}{12}$ $\frac{8}{12}$

Steps to Convert a Repeating Decimal to a Fraction in simplest form:

* Let n = the fraction for the repeating decimal
  (For $0.\overline{36}$ write n = 0.363636...
* The number of digits that repeat tells you what power of ten to multiply by. One repeating digit? Multiply by 10
  Two repeating digits? Multiply by 100
  Three repeating digits? Multiply by 1000 and so forth
  (In $0.\overline{36}$ there are two repeating digits so multiply by 100)
* Find 100 times n and write like this:

$$100n = 36.363636\ldots$$
$$n = 0.363636\ldots$$

* Subtract on both sides of the equation. Subtract bottom equation from top equation like this:

$$100n = 36.363636\ldots$$
$$-1n = 0.363636\ldots$$
$$99n = 36$$

Think: 100n minus 1n is 99n Subtract decimal numbers

* Solve the resulting one-step equation

$$99n = 36$$
$$\frac{99n}{99} = \frac{36}{99}$$
$$1n = \frac{36}{99}$$

Simplify fraction if possible

$$n = \frac{4}{11}$$

* So the simplified fraction that is equivalent to $0.\overline{36}$ is $\frac{4}{11}$

DO NOT SKIP steps in this process!!

How do we find the fraction for a repeating decimal we have not memorized?

Example: $0.\overline{39}$

$$
\begin{aligned}
100n &= 39.3939\ldots \\
-\quad n &= 0.3939\ldots \\
\hline
99n &= 39 \\
\frac{99n}{99} &= \frac{39}{99} \\
n &= \frac{39 \div 3}{99 \div 3} \qquad n = \frac{13}{33}
\end{aligned}
$$

How do we find the fraction for a repeating decimal we have not memorized?

Example: $0.\overline{342}$

$$
\begin{aligned}
1000n &= 342.342342\ldots \\
-\quad n &= 0.342342\ldots \\
\hline
999n &= 342 \\
\frac{999n}{999} &= \frac{342}{999} \\
n &= \frac{342 \div 9}{999 \div 9} = \frac{38}{111}
\end{aligned}
$$

How do we find the fraction for a repeating decimal we have not memorized?

Example: $0.2\overline{4}$

$$
\begin{aligned}
10n &= 2.4444\ldots \\
-\quad n &= 0.2444\ldots \\
\hline
9n &= 2.2 \\
\frac{9n}{9} &= \frac{2.2}{9} \\
n &= \frac{2.2 \times 10}{9 \times 10} = \frac{22}{90} = \frac{11}{45}
\end{aligned}
$$

How do we find the fraction for a repeating decimal we have not memorized?

Example: $0.4\overline{32}$

$$
\begin{aligned}
100n &= 43.23232\ldots \\
-\quad n &= 0.43232\ldots \\
\hline
99n &= 42.8 \\
\frac{99n}{99} &= \frac{42.8}{99} \\
n &= \frac{42.8}{99} = \frac{428}{990} = \frac{214}{495}
\end{aligned}
$$